TORUS-EQUIVARIANT VECTOR BUNDLES AND STABLE VECTOR BUNDLES

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Introduction.

Let T be an n-dimensional algebraic torus. Then T naturally acts on an n-dimensional projective space $\mathbf{P}^n$ and $\mathbf{P}^n$ becomes a toric variety. Let E be a vector bundle on $\mathbf{P}^n$. We say that E is torus-equivariant if there exists an isomorphism $\varphi_t : t^*E \longrightarrow E$ for every t in T where $t : \mathbf{P}^n \longrightarrow \mathbf{P}^n$ is an action of t on $\mathbf{P}^n$. We know the indecomposable torus-equivariant vector bundles on $\mathbf{P}^n$ [3]. In this paper we consider dim $\mathrm{H}^0(\mathbf{P}^n, E)$ for an indecomposable torus-equivariant vector bundle E of rank n on $\mathbf{P}^n$ (Theorem 3.6). Furthermore we have the conditions such that an indecomposable torus-equivariant vector bundle of rank three on $\mathbf{P}^3$ is a stable vector bundle (Theorem 4.1).

§1. T-equivariant vector bundles.

Let N be a free $\mathbf{Z}$-module of rank n. Let M be a dual $\mathbf{Z}$-module of N. Then there is a natural $\mathbf{Z}$-bilinear map :

$$\langle \ , \ \rangle : M \times N \longrightarrow \mathbf{Z}.$$

It can naturally be extended to $M_{\mathbf{R}} \times N_{\mathbf{R}} \to \mathbf{R}$ where $M_{\mathbf{R}} = M \otimes_{\mathbf{Z}} \mathbf{R}$ and $N_{\mathbf{R}} = N \otimes_{\mathbf{Z}} \mathbf{R}$. We denote $\varphi(\xi) = \langle \xi , \varphi \rangle$ for ξ in $M_{\mathbf{R}}$ and φ in $N_{\mathbf{R}}$. Let $T = T_N$ be an n-dimensional algebraic torus over an algebraically closed field K defined by N, i. e.

$$T_N = N \otimes_{\mathbf{Z}} K^* = Hom_{\mathbf{Z}}(M, K^*)$$

where K^* is the multiplicative group of K. Let ξ be in M then we define a homomorhism

$$e(\xi) : T \longrightarrow K^*$$

by $e(\xi)(t) = t(\xi)$ for t in T. Then we can identify M with the additive group of characters of T. Let $X = T_N\mathrm{emb}(\Delta)$ be a smooth complete toric variety of dimension n defined by a fan Δ of N for which the reader is referred to [4 , 5 , 6]. Let $U_C = \mathrm{Spec}(A_C)$ be a T-stable affine open corresponding to the n-dimensional cone C in $\Delta(n)$ where $A_C = K[C^* \cap M]$. For ξ in M, we denote by $A(\xi)$ the M-graded A-free module of rank one defined by $A(\xi)_\eta = A_{\xi+\eta}$. Let $E = E(m, P)$ be a T-equivariant vector bundle of rank r on X defined by (m, P). Then we have

$$E(U_C) = \bigoplus_{i=1}^{r} A_C(-\xi(C)_i)$$

for some characters $\xi(C)_1, \ldots, \xi(C)_r$ in M (see [2]). Let $V = \bigoplus_{i=1}^{r} K[M]$ be a K-vector space and let $h_{C,C'}$ be the K-automorphism of V defined by $P(C,C')$ for C and C' in $\Delta(n)$, i. e.

$$h_{C,C'}(x_1,\ldots,x_r) = (x_1,\ldots,x_r)P(C,C').$$

Let

$$V_C = \{(x_1,\ldots,x_r) \in V \mid x_i \in A_C(-\xi(C)_i) \text{ for } 1 \leq i \leq r\}$$

be a subspace of V. Then we can identify V_C with $E(U_C)$ as a vector space. We denote by $|C|$ the set of fundamental generators of a cone C Then we have :

LEMMA 1. 1. *Let $x = (x_1,\ldots,x_r)$ be in V and put $x_i = \Sigma p_{ij} e(\xi_{ij})$ $(p_{ij} \neq 0)$ for $1 \leq i \leq r$. Then, for any cone C in $\Delta(n)$, the following are equivalent :*

(1) $x \in V_C$.

(2) $\varphi(\xi_{ij}) \geq \varphi(-\xi(C)_i)$ *for every j and φ in $|C|$ $(1 \leq i \leq r)$.*

Proof. This is easy conclusion from the definition of $A_C(-\xi(C)_i)$ and V_C.

PROPOSITION 1. 2. *Let E be a T-equivariant vector bundle defined by (m, P) and let $h_{C,C'}$ be a K-automorphism defined by $P(C, C')$. Let C be a cone in $\Delta(n)$. Then*

$$H^0(X, E) = \{x_C \in V_C \mid h_{C,C'}(x_C) \in V_{C'} \text{ for every } C' \text{ in } \Delta(n)\}$$

Proof. Let s be in $H^0(X, E)$. Then $x_C = s \mid U_C \in E(U_C) = V_C$. By the construction of E (see [2]) we have a commutative diagram of vector spaces

$$\begin{array}{ccccc} E(U_C) & \longrightarrow & A_{C\cap C'} \otimes E(U_C) & \longrightarrow & V \\ & & \Big\downarrow f_{C,C'} & & \Big\downarrow h_{C,C'} \\ E(U_{C'}) & \longrightarrow & A_{C\cap C'} \otimes E(U_{C'}) & \longrightarrow & V \end{array}$$

for every C and C' in $\Delta(n)$, where $f_{C,C'}$ is a patching function of E defined by $P(C,C')$. It is easy to see that $x_{C'} = s|U_{C'} = f_{C,C'}(x_C)$ i.e. $h_{C,C'}(x_C)$ is in $V_{C'}$. Conversely let x_C be in V_C such that $x_C = h_{C,C'}(x_C)$ is in V_C, for every C' in $\Delta(n)$, then $\{x_C | C \in \Delta(n)\}$ can be patched up and defines a global section of E.

§2. Subsets of M.

In this section we show some lemmas and these results will be used in the next section. Now we first use conbinatorical result.

LEMMA 2. 1. *Let b be an integer. Then the number of integral solutions of*

$$x_1 + \cdots + x_n \leq b, \quad x_1 \geq 0, \ldots, x_n \geq 0$$

is

$$g(b) = \begin{cases} \frac{1}{n!}(b+1)(b+2)\dots(b+n) & \text{if } b \geq 0, \\ 0 & \text{if } b < 0. \end{cases}$$

DEFINITION 2.2. Let A be a finite subset of M. we denote by $v(A)$ the number of elemens of A.

Let $\{\varphi_1, \dots, \varphi_n\}$ be a $\mathbf{Z}$-base of N and put $\varphi_0 = -\varphi_1 - \cdots - \varphi_n$.

LEMMA 2.3. *Let $b_0, \dots, b_n$ be integers and put*

$$A = \{\xi \in M \mid \varphi_i(\xi) \geq b_i \ \text{for } 0 \leq i \leq n\}.$$

Then we have $v(A) = g(-b_0 - \cdots - b_n)$ where $g(l)$ is defined in Lemma 2.1.

Proof. Put $x_i = \varphi_i(\xi) \in \mathbf{Z}$ for ξ in M $(1 \leq i \leq n)$. Then ξ is in A if and only if $x_1 + \cdots + x_n \leq -b_0$ and $x_i \geq b_i$ $(1 \leq i \leq n)$. Put $y_i = x_i - b_i$ $(1 \leq i \leq n)$. Then we have

$$y_1 + \cdots + y_n \leq -b_0 - \cdots - b_n \text{ and } y_i \geq 0 \ (1 \leq i \leq n).$$

Hence, by Lemma 2.1 we have $v(A) = g(-b_0 - \cdots - b_n)$.

DEFINITION 2.4. Let $a_0, \dots, a_n$ be positive integers and l be an integer. Put

$$M_i = \left\{ \xi \in M \mid \varphi_0(\xi) \geq a_0 - l, \ \varphi_k(\xi) \geq \begin{cases} a_i & \text{if } k = i \\ 0 & \text{if } k \neq i \end{cases} \right\},$$

$$G_{ij} = \left\{ \xi \in M \mid \varphi_0(\xi) \geq -l, \ \varphi_k(\xi) \geq \begin{cases} a_i & \text{if } k = i \\ a_j & \text{if } k = j \\ 0 & \text{otherwise} \end{cases} \right\},$$

$$M_{ij} = \{\xi \in G_{ij} \mid \varphi_0(\xi) < a_0 - l\},$$

Let $k_1, \dots, k_t$ be integers such that $0 < k_1 < \dots < k_t < \min(i,j)$. Put

$$M_{ij}(k_1, \dots, k_t) = \{\xi \in M_{ij} \mid \varphi_{k_s}(\xi) \geq a_{k_s} (1 \leq s \leq t)\}, \quad \overline{M}_{ij} = \bigcup_{0<k<\min(i,j)} M_{ij}(k).$$

LEMMA 2.5. *Assume that $0 < i < j$, then we have*

(1) $v(M_i) = g(l - a_0 - a_i)$.

(2) $v(M_{ij}) = g(l - a_i - a_j) - g(l - a_0 - a_i - a_j)$.

(3) $v(M_{ij}(k_1, \dots, k_t)) = g(l - a_{k_1} - \dots - a_{k_t} - a_i - a_j) - g(l - a_0 - a_{k_1} - \dots - a_{k_t} - a_i - a_j)$.

(4) $v(\overline{M}_{ij}) = = \sum_{0<k_1<i} v(M_{ij}(k_1)) - \sum_{0<k_1<k_2<i} v(M_{ij}(k_1, k_2)) + \sum_{0<k_1<k_2<k_3<i} v(M_{ij}(k_1, k_2, k_3)) - \dots$

Proof. These assertions are easy conclusion by applying Lemma 2.3 for each case. So we ommit.

By Lemma 2.5 we have the following lemma.

LEMMA 2.6. $\sum_{i>0} v(M_{ij}) + \sum_{0<i<j} v(M_{ij}) - \sum_{0<i<j} v(\overline{M}_{ij})$

$$= \sum_{0<i_1<i_2} g(l-a_{i_1}-a_{i_2}) - \sum_{0<i_1<i_2<i_3} g(l-a_{i_1}-a_{i_2}-a_{i_3}) + \sum_{0<i_1<i_2<i_3<i_4} g(l-a_{i_1}-a_{i_2}-a_{i_3}) - \dots$$

§3. Case of projective spaces.

In this section we consider the case of $X = \mathbf{P}^n$ and a T-equivariant vector bundle E of rank n. Let $\{\varphi_1,\dots,\varphi_n\}$ be a $\mathbf{Z}$-base of N and put $\varphi_0 = -\varphi_1-\dots-\varphi_n$. Let $\mathbf{P}^n = T_N \operatorname{emb}(\Delta)$ be a toric variety defined by a fan Δ where the set of n-dimensional cones of Δ is

$$\Delta(n) = \{C_i \mid C_i = \sum_{j\neq i} \mathbf{R}_0\varphi_j \quad (0 \leq i \leq n)\}.$$

Let E be a T-equivariant vector bundle defined by the exact sequence :

$$0 \longrightarrow O_{\mathbf{P}^n} \longrightarrow \bigoplus_{i=0}^{n} O_{\mathbf{P}^n}(a_i) \longrightarrow E^* \longrightarrow 0$$

for positive integers a_i. Then we may assume that $E(l)$ is defined by the follwing (m, P) :

$$m_{C_j}(\varphi_i) = (0,\dots,0, \overset{i\text{-th}}{a_i}, 0,\dots,0) \quad \textit{for} \quad 1 \leq i \leq n,\ 1 \leq j \leq n,\ i \neq j,$$

$$m_{C_i}(\varphi_0) = (-l,\dots,-l, \overset{i\text{-th}}{a_0-l}, -l,\dots,-l) \quad \textit{for} \quad 1 \leq i \leq n$$

and

$$P(C_0, C_i) = \begin{array}{c} \qquad\qquad\quad i\text{-}th \\ \begin{pmatrix} 1 & & & & 1 & & & \\ & 1 & & & \vdots & & & \\ & & \ddots & & \vdots & & & \\ & & & \ddots & \vdots & & & \\ & & & & 1 & & & \\ & & & & \vdots & \ddots & & \\ & & & & \vdots & & \ddots & \\ & & & & 1 & & & 1 \end{pmatrix} \end{array}.$$

Let $x = (p_1 e(\xi_1), \dots, p_n e(\xi_n))$ be in V_{C_0}. Then we have

$$h_{C_0,C_i}(x) = (p_1 e(\xi_1), \dots, \overset{i\text{-th}}{\sum_{k=1}^{n} p_k e(\xi_k)}, \dots, p_n e(\xi_n))$$

for $1 \leq i \leq n$ where h_{C_0,C_i} is a K-automorphism of V defined in §1.

DEFINITION 3.1 For ξ in M we define

$$s_i(\xi) = (0,\dots,0, \overset{i\text{-th}}{e(\xi)}, 0,\dots,0) \quad (1 \leq i \leq n),$$

$$s_{ij}(\xi) = (0,\dots0, \overset{i\text{ th}}{e(\xi)}, 0,\dots, 0, \overset{j\text{-th}}{-e(\xi)}, 0,\dots,0) \quad (1 \leq i < j \leq n)$$

and

$$W = \{s \in V_{C_0} \mid h_{C_0,C_i}(s) \in V_{C_i} \text{ for } 1 \leq i \leq n\}.$$

LEMMA 3.2.

(1) $s_i(\xi) \in W$ *if and only if* $\xi \in M_i$,

(2) $s_{ij}(\xi) \in W$ *if and only if* $\xi \in G_{ij}$

where M_i *and* G_{ij} *are defined in Definition* 2.4.

Proof. we first prove (1). It is easy that $s_i(\xi) \in$ W if and only if $s_i(\xi) \in V_{C_0}$ and $h_{C_0,C_j}(s_i(\xi)) \in V_{C_j}$ for $1 \leq j \leq n$. These means, by Definition 2.4, that

$$\varphi_0(\xi) \geq a_0 - l, \quad \varphi_j(\xi) \geq \begin{cases} a_i & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases} \qquad (1 \leq j \leq n)$$

i.e. ξ is in M_i. The second assertion is proved by the same way.

LEMMA 3.3. $\{s_i(\xi) | \xi \in M_i\}$ $(1 \leq i \leq n)$ *and* $\{s_{ij}(\xi) \mid \xi \in G_{ij}\}$ $(1 \leq i < j \leq n)$ *generate* W *as a vector space.*

Proof. we may only prove that $x = (p_1 e(\eta_n), \ldots, p_n e(\eta_n))$ in W is generated by s_i and s_{ij}. Put $x = x_1 + \ldots + x_k$ where

$$x_i = (p_{i1} e(\xi_i), \ldots, p_{in} e(\xi_i)) \quad (1 \leq i \leq k)$$

and $\xi_i \neq \xi_j$ if $i \neq j$. It is easy to see that x is in W if and only if x_i is in W for $1 \leq i \leq k$. Hence we may assume

$$x = (p_1 e(\xi), \ldots, p_n e(\xi)).$$

Now assume $p_1 + \ldots + p_n \neq 0$ then the condition $h_{C_0,C_i}(x) \in V_{C_i}$ *for* $0 \leq i \leq n$ means that

$$\varphi_0(\xi) \geq a_0 - l, \text{ and } \varphi_i(\xi) \leq a_i \quad (1 \leq i \leq n).$$

Hence we have $x = p_1 s_1(\xi) + \ldots + p_n s_n(\xi)$. Assume $p_1 + \ldots + p_n = 0$ then the condition $h_{C_0,C_i}(x) \in V_{C_i}$ for $0 \leq i \leq n$ means that

$$\varphi_0(\xi) \geq -l, \text{ and } \varphi_i(\xi) \geq \begin{cases} a_i & \text{if } p_i \neq 0 \\ 0 & \text{if } p_i = 0 \end{cases} \qquad (1 \leq i \leq n).$$

we denote non-zero p_i by $p_{i_1}, \ldots, p_{i_t}$, then we have

$$x = p_{i_1} s_{i_1 i_t}(\xi) + \ldots + p_{i_{t-1}} s_{i_{t-1} i_t}(\xi).$$

Therefore W is generated by s_i and s_{ij}.

LEMMA 3.4.

(1) *Assume that* ξ *is in* $G_{ij} - M_{ij}$ *and* $\varphi_0(\xi) \geq a_0 - l$ *for some* i, j $(i < j)$, *then we have that* $s_i(\xi)$ *and* $s_j(\xi)$ *are in* W *and* $s_{ij}(\xi) = s_i(\xi) - s_j(\xi)$.

(2) *Assume that* ξ *is in* $M_{ij} \cap M_{ki} \cap M_{kj} = M_{ij}(k)$ *for some* $0 < k < i < j$, *then we have*

that $s_{ij}(\xi)$, $s_{ki}(\xi)$, *and* $s_{kj}(\xi)$ *are in* W *and* $s_{ij}(\xi) = s_{kj}(\xi) - s_{ki}(\xi)$

Proof. By Lemma 3.2 and Definition 2.4, the assertions of the lemma are true.

By Lemma 3.4, we can consider $\{s_i(\xi) | \xi \in M_i\}$ $(1 \leq i \leq n)$ and $\{s_{ij}(\xi) | \xi \in \overline{M}_{ij}\}$ $(1 \leq i < j \leq n)$ as generators of W.

LEMMA 3.5. $\{s_i(\xi) | \xi \in M_i\}$ $(1 \leq i < j \leq n)$ *and* $\{s_{ij}(\xi) | \xi \in M_{ij} - \overline{M}_{ij}\}$ $(1 \leq i < j \leq n)$ *are linearly independent.*

Proof. We consider the linear combination of s_i and s_{ij}. Decompose it into same degree then we may prove that the linear combination

$$\sum_i a_i s_i(\xi) + \sum_{i<j} b_{ij} s_{ij}(\xi) = 0$$

such that ξ is in M_i if $a_i \neq 0$ and ξ is in $M_{ij} - \overline{M}_{ij}$ if $b_{ij} \neq 0$, imply $a_i = 0$ and $b_{ij} = 0$ for all i, j. We prove first all $a_i = 0$. Assume $a_k \neq 0$ for some k then there exists h such that $b_{kh} \neq 0$ $(k < h)$ or $b_{hk} \neq 0$ $(h < k)$. If $h < k$ then interchange h with k so we may assume $b_{kh} \neq 0$ $(k < h)$. Hence ξ is in M_k and in $M_{kh} - \overline{M}_{kh}$. This means, by the definition of M_k and M_{kh}, that $\varphi_0(\xi) \geq a_0 - l$ and $\varphi_0(\xi) < a_0 - l$, a contradiction. Hence all $a_i = 0$. Now assume that $\sum b_{ij} s_{ij}(\xi) = 0$ and ξ is in $M_{ij} - \overline{M}_{ij}$ if $b_{ij} \neq 0$. Suppose that $b_{kh} \neq 0$ for some k and h $(k < h)$. Since k-th and h-th components of $s_{kh}(\xi)$ are not zero, there exists t $(t \neq k, h)$ such that $b_{ht} \neq 0$ $(t > h)$ or $b_{th} \neq 0$ $(t < h)$. In case $t > h$, we have ξ is in M_{kh} and in M_{ht} hence ξ is in $M_{ht}(k) \subset \overline{M}_{ht}$ a contradiction. In case $k < t < h$ (or $t < k$) we have, by the same way, that ξ is in $M_{th}(k) \subset \overline{M}_{th}$ (or $M_{kh}(t) \subset \overline{M}_{kh}$) a contradiction. Therefore we have all $b_{ij} = 0$.

THEOREM 3.6. *Let E be vector bundle of rank n on $\mathbf{P}^n$ defined by the exact sequence*

$$0 \longrightarrow O_{\mathbf{P}^n} \xrightarrow{f} \bigoplus_{i=0}^{n} O_{\mathbf{P}^n}(a_i) \longrightarrow E^* \longrightarrow 0$$

such that f sends 1 to $(X_0^{a_0},\ldots,X_n^{a_n})$ where $X_0,\ldots,X_n$ are homgeneous coordinates of $\mathbf{P}^n$ and a_i are positive integers. Then we have

$$\dim \mathrm{H}^0(\mathbf{P}^n, E(l)) = \sum_{i<j} g(l - a_i - a_j) - \sum_{i<j<k} g(l - a_i - a_j - a_k)$$
$$+ \sum_{i<j<k<h} g(l - a_i - a_j - a_k - a_h) - \ldots$$

where

$$g(b) = \begin{cases} \frac{1}{n!}(b+1)(b+2)\cdots(b+n) & \text{if } b \geq 0, \\ 0 & \text{if } b < 0. \end{cases}$$

Proof. By Proposition 1.2, dim $\mathrm{H}^0(\mathbf{P}^n, E(l))$ = dim W. Since we know the base of W, we have the conclusion by Lemma 2.6.

§4. Stable vector bundles of rank three on $\mathbf{P}^3$.

THEOREM 4.1. *Let E be a vector bundle of rank three on $\mathbf{P}^3$ defined by the exact sequence*

$$0 \longrightarrow O_{\mathbf{P}^3} \longrightarrow \bigoplus_{i=0}^{3} O_{\mathbf{P}^3}(a_i) \longrightarrow E^* \longrightarrow 0$$

for positive integers a_i. Assume that $0 < a_0 \leq a_1 \leq a_2 \leq a_3$ then the following are equivalent :

(1) *E is a stable vector bundle.*

(2) $2a_3 < a_0 + a_1 + a_2$ *and* $a_2 + a_3 < 2(a_0 + a_1)$.

Proof. Since E is a vector bundle of rank three on $\mathbf{P}^3$, we know that E is stable if and only if $\mathrm{H}^0(\mathbf{P}^3, E_{norm}) = 0$ and $\mathrm{H}^0(\mathbf{P}^3, (E^*)_{norm}) = 0$ where E_{norm} and $(E^*)_{norm}$ are normalized vector bundles. From the exact sequence

$$0 \longrightarrow O_{\mathbf{P}^3}(-l) \longrightarrow \bigoplus_{i=0}^{3} O_{\mathbf{P}^3}(a_i - l) \longrightarrow E^*(-l) \longrightarrow 0$$

we have

$$0 \longrightarrow \mathrm{H}^0(\mathbf{P}^3, O_{\mathbf{P}^3}(-l)) \longrightarrow \bigoplus_{i=0}^{3} \mathrm{H}^0(\mathbf{P}^3, O_{\mathbf{P}^3}(a_i - l)) \longrightarrow H^0(\mathbf{P}^3, E^*(-l)) \longrightarrow 0.$$

Hence $\mathrm{H}^0(\mathbf{P}^3, E^*(-l)) = 0$ if and only if $a_3 - l < 0$.

Put $\alpha = (a_0 + a_1 + a_2 + a_3) \bmod 3$, $k = (a_0 + a_1 + a_2 + a_3 - \alpha)/3$ and

$$\beta = \begin{cases} 0 & \text{if } \alpha = 0, \\ 2 & \text{if } \alpha = 1, \\ 1 & \text{if } \alpha = 2. \end{cases}$$

Put $l = (a_0 + a_1 + a_2 + a_3 + \beta)/3 = (3k + \alpha + \beta)/3$ then $E^*(-l)$ is a normalized vector bundle. Hence

$$\begin{aligned} \mathrm{H}^0(\mathbf{P}^3, E^*(-l)) = 0 &\Longleftrightarrow a_3 < l \\ &\Longleftrightarrow 3a_3 < 3k + \alpha + \beta \\ &\Longleftrightarrow 3a_3 < 3k + \alpha \\ &\Longleftrightarrow 2a_3 < a_0 + a_1 + a_2. \end{aligned}$$

Now we see the another condition such that $\mathrm{H}^0(\mathbf{P}^3, E_{norm}) = 0$. Since $g(x)$ in §2 is not decreasinig function, by Theorem 3.6, that $\mathrm{H}^0(\mathbf{P}^3, E(l)) = 0$ if and only if $l - a_0 - a_1 < 0$. Put $\alpha = (a_0 + a_1 + a_2 + a_3) \bmod 3$ and $l = (a_0 + a_1 + a_2 + a_3 - \alpha)/3$ then $E(l)$ is a normalized vector bundle. Hence

$$\begin{aligned} \mathrm{H}^0(\mathbf{P}^3, E(l)) = 0 &\Longleftrightarrow l < a_0 + a_1 \\ &\Longleftrightarrow 3l < 3(a_0 + a_1) \\ &\Longleftrightarrow 3l + \alpha < 3(a_0 + a_1) \end{aligned}$$

$$\Longleftrightarrow \quad a_2 + a_3 < 2(a_0 + a_1).$$

Therefore we have the conclusion.

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