

Paradoxical Results in a Lobbying Model of Protection

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Okuguchi and Yamazaki (1998) generalized Long and Soubeyran's lobbying model of protection. They also constructed a simple numerical example with the linear inverse demand and cost functions to illustrate a paradoxical result in which entry of a new domestic firm leads to an increase in the domestic incumbents' profits. This paper shows that the paradoxical result occurs even if cost functions are quadratic. Numerical examples in this paper, together with the one in Okuguchi and Yamazaki (1998), indicate that the paradoxical result is more likely if the number of domestic firms is small and if the number of foreign firms is large.

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1. Introduction

There are many cases that economic policies are determined through lobbying processes. In some countries, lobbying plays a significant role in determining the tariff rate as an economic policy. Moore and Suranovic (1993) and Long and Soubeyran (1996) analyzed lobbying in an imperfect competitive market where domestic firms compete with foreign firms in the domestic market. Okuguchi and Yamazaki (1998) generalized a lobbying model formulated by Long and Soubeyran (1996). In oligopoly without lobbying for protection, the domestic incumbents' profits decrease if entry of a new domestic firm occurs. In the presence of lobbying, however, entry may lead to a higher tariff rate, which in turn may enable the domestic incumbents to get larger profits than before the entry. Okuguchi and Yamazaki (1998) constructed a model with the linear inverse demand and cost functions, and pointed out that such a paradox occurs under specific parameter values. They also concluded that the paradoxical result is more likely if the number of domestic firms is small and if, in addition, the number of foreign firms is large. However, their conclusion may depend on the linearity of the model. The aim of this paper is to show that their conclusion still holds even if we introduce non-linearity in the model.

Section 2 will briefly review the generalized Long and Soubeyran's lobbying model in Okuguchi and Yamazaki (1998). In Section 3, we will analyze the effects of entry of a domestic firm and show that paradoxical results could occur in a model with non-linear cost functions. Section 4 concludes.

2. Model

Suppose n domestic and n^* foreign firms producing one identical good compete in the domestic market, and let x_i and x_j^* be i -th domestic and j -th foreign firms' outputs, respectively. Furthermore, let $p = f(\sum x_i + \sum x_j^*)$ be the inverse demand function, where p is the price of the good produced by domestic and foreign firms, and $C_i(x_i)$ and $C_j^*(x_j^*)$ be i -th domestic and j -th foreign firms' cost functions, respectively. If t is the specific tariff rate imposed on the foreign product by the home country, domestic and foreign firms' profits π_i and π_j^* are defined as

$$(1) \quad \pi_i \equiv x_i f(Q) - C_i(x_i), i = 1, 2, \dots, n$$

and

$$(2) \quad \pi_j^* \equiv x_j^* f(Q) - C_j^*(x_j^*) - tx_j^*, j = 1, 2, \dots, n^*$$

respectively, where $Q = \sum_{i=1}^n x_i + \sum_{j=1}^{n^*} x_j^*$ is the total demand in the home country. All firms are assumed to form expectations on all other rivals' output à la Cournot. We now introduce

Assumption 1:

$$(A.1) \quad f' < C_i''(x_i), i = 1, 2, \dots, n, f' < C_j^{*''}(x_j^*), j = 1, 2, \dots, n^*,$$

$$(A.2) \quad f' + x_i f'' < 0, i = 1, 2, \dots, n, f' + x_j^* f'' < 0, j = 1, 2, \dots, n^*.$$

“A” before a number in a parenthesis refers to an assumption. (A.1) and (A.2) are generally made in the literature on Cournot oligopoly (see Okuguchi (1976), and Okuguchi and Szidarovszky (1990)). Under Assumption 1, the first-order condition for the domestic firm i 's profit maximization implies that each domestic firm's output is a function of the industry output.

$$(3) \quad x_i = \varphi_i(Q), i = 1, 2, \dots, n.$$

Similarly, the first-order condition for the foreign firm j 's profit maximization implies

$$(4) \quad x_j^* = \varphi_j^*(Q, t), i = 1, 2, \dots, n^*.$$

Furthermore, Assumption 1 ensures that $\varphi_i(Q)$ is decreasing in Q and that $\varphi_j^*(Q, t)$ is decreasing in Q and t (See Okuguchi and Yamazaki (1998)). The Cournot equilibrium industry output is a solution of the equation

$$(5) \quad Q = \sum_{i=1}^n \varphi_i(Q) + \sum_{j=1}^{n^*} \varphi_j^*(Q, t) \equiv \Phi(Q, t).$$

The following theorem is due to Okuguchi and Yamazaki (1998).

Theorem 1: Under Assumption 1, there exists a unique Cournot-Nash equilibrium at each second-stage game. Furthermore, the equilibrium profits of all domestic firms increase if the tariff rate increases.

Proof: See Okuguchi and Yamazaki (1998).

This theorem generalizes Lemma 1 in Long and Soubeyran (1996, p.22).

In stage 1, domestic firms engage in lobbying for protection. The tariff rate t is determined as a result of domestic firms' lobbying activities for protection. Let the direct lobbying cost of firm

i be y_i and indirect one $I_i(y_i)$. Then $Y = \sum_{i=1}^n y_i$ is the total lobbying expenditure by all

domestic firms. We are interested in a case where the tariff rate t depends on the total lobbying expenditure; $t = t(Y)$. Following Long and Soubeyran (1996), we assume

Assumption 2:

$$(A.3) \quad I_i' > 0, \quad I_i'' > 0$$

and

$$(A.4) \quad t' > 0, \quad t'' \leq 0.$$

Since all firms behave as Cournot oligopolists in stage 2 of output determination, firm i 's profit in stage 1, Π_i , is given by

$$(6) \quad \Pi_i \equiv \pi_i(Q(t)) - y_i - I_i(y_i), i = 1, 2, \dots, n.$$

Assume all domestic firms to form expectations on all rivals' direct lobbying cost à la Cournot.

The first-order condition for maximizing (5) with respect to y_i yields

$$\begin{aligned} (7) \quad & \frac{\partial \Pi_i}{\partial y_i} \\ & = Q'(t(Y))t'(Y)\varphi_i(Q(t(Y)))f'(Q(t(Y))(1 - \varphi_i'(Q(t(Y)))) - 1 - I_i'(y_i) \\ & \equiv J_i(Y) - 1 - I_i'(y_i) \\ & \equiv G_i(Y, y_i) = 0, i = 1, 2, \dots, n, \end{aligned}$$

where

$$(8) \quad J_i'(Y) = Q'(t(Y))t'(Y)\varphi_i(Q(t(Y)))f'(Q(t(Y))) \\ \times \frac{2f'(Q(t(Y))) + \varphi_i(Q(t(Y)))f''(Q(t(Y))) - C_i''(Q(t(Y)))}{f'(Q(t(Y))) - C_i''(Q(t(Y)))} \\ > 0, i = 1, 2, \dots, n.$$

In general, the sign of J_i' is indeterminate. However, we assume

Assumption 3:

$$(A.5) \quad J_i'(Y) > 0, i = 1, 2, \dots, n.$$

and

$$(A.6) \quad J_i'(Y) < I_i''(y_i), i = 1, 2, \dots, n.$$

If $C_i'' = C_j'' = 0$ as in Long and Soubeyran (1996) and if, in addition, $f'' = 0$ and $t'' = 0$, (A.5) is satisfied. As we shall see in Section 3, (A.5) still holds even if the cost functions are quadratic. (A.6) ensures the second-order condition for maximizing (6) with respect to y_i .

Solving (7) with respect to y_i and taking into account (A.3) and (A.5),

$$(8) \quad y_i \equiv \Psi_i'(Y), i = 1, 2, \dots, n,$$

where

$$(9) \quad \frac{d\Psi_i}{dY} = \frac{J_i'}{I_i''} > 0, i = 1, 2, \dots, n.$$

The first-stage equilibrium total expenditure is identical to the solution of the equation

$$(10) \quad Y = \sum_{i=1}^n \Psi_i(Y) \equiv \Psi(Y).$$

Finally, we assume

Assumption 4:

$$(A.7) \quad \frac{d\Psi}{dY} < 1$$

$$(A.8) \quad \Psi_i(0) > 0.$$

Assumptions (A.5), (A.6), (A.7) and (A.8) may seem restrictive. However, Section 4 in Okuguchi and Yamazaki (1998) showed that this is not so. Now we state

Theorem 2: Under Assumptions 1, 2, 3 and 4, the generalized Long and Soubeyran's two-stage lobbying game has a unique subgame perfect equilibrium.

Proof: See Okuguchi and Yamazaki (1998).

We have described the model. Under Assumptions 1, 2, 3 and 4, a unique subgame perfect equilibrium is well determined. The next section will examine the effects of entry of a domestic firm on the domestic incumbent's profit.

3.Paradoxical Results

This section will analyze a paradox that the domestic incumbents' profit increases if entry occurs. In general, the total derivative of domestic firm's profit with respect to the number of domestic firms can be decomposed as

$$(11) \quad \frac{d\pi_i}{dn} = \frac{\partial \pi_i}{\partial t} \frac{\partial t}{\partial n} + \frac{\partial \pi_i}{\partial n}.$$

The second term on the right hand side of (11) is normally negative. If the entry of a domestic firm increases the total direct lobbying cost Y , then the first term on the right hand side of (11) is positive by Theorem 1. If the first term dominates the second term, the total derivative $\frac{d\pi_i}{dn}$ is positive. This is the case where a paradox occurs. Okuguchi and Yamazaki (1998) constructed a model with the linear inverse demand and cost functions and pointed out that such a paradox occurs under specific parameter values. The paradoxical result might occur because of linearity in the model. It is known that non-linearity sometimes violates an economic conclusion which holds under a linear model. The aim of this section is to show that the paradoxical results still hold in a non-linear model.

Consider the following linear inverse demand and quadratic cost functions.

$$(12) \quad f(Q) = a - bQ$$

$$(13.1) \quad C_i(x_i) = c_{i1}x_i^2 + c_{i2}x_i, i = 1, 2, \dots, n,$$

$$(13.2) \quad C_j^*(x_j^*) = c_{j1}^*(x_j^*)^2 + c_j^*x_j^*, j = 1, 2, \dots, n^*.$$

By simple calculations

$$(3') \quad x_i = \varphi_i(Q) \equiv \frac{a - c_{i2}}{b + 2c_{i1}} - \frac{b}{b + 2c_{i1}} Q, i = 1, 2, \dots, n,$$

$$(4') \quad x_j^* = \varphi_j^*(Q, t) \equiv \frac{a - c_{j2}^* - t}{b + 2c_{j1}^*} - \frac{b}{b + 2c_{j1}^*} Q, j = 1, 2, \dots, n^*.$$

Consider the symmetric case where

$$(A.9) \quad c_{i1} = c_{j1}^* \equiv c_1 \quad \text{and} \quad c_{i2} = c_{j2}^* \equiv c_2, i = 1, 2, \dots, n, j = 1, 2, \dots, n^*.$$

The equilibrium condition for the industry output yields

$$(14) \quad Q(t) = \frac{(n + n^*)(a - c_2) - n^* t}{2c_1 + (1 + n + n^*)b},$$

Substitution of (14) into (12) gives the second-stage equilibrium price. Then we get

$$(15.1) \quad p - c_2 = \frac{(2c_1 + b)(a - c_2) + n^* bt}{2c_1 + (1 + n + n^*)b},$$

$$(15.2) \quad p - c_2 - t = \frac{(2c_1 + b)(a - c_2 - t) - nbt}{2c_1 + (1 + n + n^*)b}.$$

Since

$$(3'') \quad x_i = \varphi_i(Q) = \frac{1}{b + 2c_1} (p - c_2), i = 1, 2, \dots, n,$$

the second stage profit for i-th domestic incumbent is

$$(16) \quad \begin{aligned} \pi_i &= (p - c_2) \left(\frac{p - c_2}{b + 2c_1} \right) - c_1 \left(\frac{p - c_2}{b + 2c_1} \right)^2 \\ &= \frac{b + c_1}{(b + 2c_1)^2} (p - c_2)^2. \end{aligned}$$

To analyze the first stage of the game, let $t(Y)$ and $I_i(y_i)$ be simplified as follows.

$$(17) \quad t(Y) = \tau Y, I_i(y_i) = \kappa y_i^2,$$

where τ and κ are positive constants. Then the first stage profit for i-th domestic incumbent becomes

$$(6') \quad \Pi_i = \frac{b + c_1}{(b + 2c_1)^2} (p - c_2)^2 - y_i - \kappa y_i^2.$$

Differentiate (17) with respect to y_i ,

$$(7') \quad \frac{\partial \Pi_i}{\partial y_i} = 2 \frac{b+c_1}{(b+2c_1)^2} (p-c_2) \frac{n^* b \tau}{2c_1 + (1+n+n^*)b} - 1 - 2\kappa y_i,$$

and

$$(18) \quad \frac{\partial^2 \Pi_i}{\partial y_i^2} = 2 \frac{b+c_1}{(b+2c_1)^2} \left(\frac{n^* b \tau}{2c_1 + (1+n+n^*)b} \right)^2 - 2\kappa = J_i' - 2\kappa.$$

Hence, (A.5) is satisfied. (A.6) is equivalent to

$$(A.6') \quad \frac{b+c_1}{(b+2c_1)^2} \left(\frac{n^* b \tau}{1+n+n^*} \right)^2 < \kappa.$$

Setting (7') equal to 0,

$$(8') \quad y_i = \Psi_i(Y) \\ = \frac{1}{\kappa} \left\{ \frac{(b+c_1)((2c_1+b)(a-c_2) + n^* b \tau) n^* b \tau}{(b+2c_1)^2 (2c_1 + (1+n+n^*)b)^2} - \frac{1}{2} \right\}$$

and

$$(19) \quad \Psi(Y) = \sum_{i=1}^n \Psi_i(Y) \\ = \frac{n}{\kappa} \left\{ \frac{(b+c_1)((2c_1+b)(a-c_2) + n^* b \tau) n^* b \tau}{(b+2c_1)^2 (2c_1 + (1+n+n^*)b)^2} - \frac{1}{2} \right\}.$$

The equilibrium total lobbying expenditure which solves $Y = \Psi(Y)$ is easily derived as

$$(20) \quad Y = \frac{(b+c_1)(b+2c_1)(a-c_2)n^* b \tau - \frac{1}{2}(b+2c_1)^2 \{2c_1 + (1+n+n^*)b\}^2}{\frac{\kappa}{n} \{2c_1 + (1+n+n^*)b\}^2 - (b+c_1)(n^* b \tau)^2}.$$

The equilibrium total lobbying expenditure determines all equilibrium values.

Since domestic firm's second-stage profit is quadratic in $p - c_2$ (See (16)), we look at

$$(21) \quad \frac{d}{dn} (p - c_2)(n) = \frac{\partial}{\partial t} (p - c_2)(n) \frac{\partial t}{\partial n} + \frac{\partial}{\partial n} (p - c_2)(n)$$

instead of (11). (21) is equivalent to

$$(21') \quad \frac{d}{dn}(p-c)(n) = \frac{n^*b}{2c_1 + (1+n+n^*)b} \tau \frac{\partial Y}{\partial n} - \frac{\{(2c_1 + b)(a - c_2) + n^*b\tau Y\}b}{(2c_1 + (1+n+n^*)b)^2}.$$

The second term on the right hand side of (21') is clearly negative.

$\frac{\partial}{\partial t}(p-c)(n) = \frac{n^*b}{2c_1 + (1+n+n^*)b}$ in (21') is the direct effect of the tariff rate. Since

$$(22) \quad \frac{\partial}{\partial n} \left(\frac{n^*b}{2c_1 + (1+n+n^*)b} \right) < 0$$

and

$$(23) \quad \frac{\partial}{\partial n^*} \left(\frac{n^*b}{2c_1 + (1+n+n^*)b} \right) = \frac{b\{2c_1 + (1+n)b\}}{\{2c_1 + (1+n+n^*)b\}^2} > 0,$$

the direct effect is large, if n is small and n^* is large. Hence, it is more likely that the first term in (21') dominates the second term, if n is small and n^* is large.

To be more specific, let

$$(24) \quad a - c_2 = 10, b = 1, \kappa = 1, \tau = 1.$$

Then Y in (20) becomes a function of only n and n^* . Numerical calculations for $n^*=4$ shows that if c_1 is less than or equal to some value between 0.010 and 0.011, domestic firms' profits increase if n increases from 1 to 2. Paradoxical results can be found for $n^*=5$. However, there is no paradox for n^* less than 4. These numerical observations confirm the conclusion: the paradox is more likely if the number of domestic firm is small and if the number of foreign firms is large.

4. Concluding Remarks

Okuguchi and Yamazaki (1998) showed how the first and second stage equilibria for lobbying game are established under more general conditions than Long and Soubeyran (1996). Okuguchi and Yamazaki (1999) also constructed a simple numerical example illustrating a paradoxical result in which entry of a new domestic firm leads to an increase in the domestic incumbents' profits. However, in the numerical example, they assumed that both domestic and foreign firms' cost functions are linear. Their paradox might be due to the linearity of the cost functions. This paper has shown that the paradox occurs even in a model with non-linear cost functions. Numerical examples in this paper, together with the one in Okuguchi and Yamazaki (1998), indicate that the paradoxical result is more likely if the number of domestic firms is small and if, in addition, the number of foreign firms is large.

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